

THE LUMINOSITY FUNCTION FOR THE GLOBULAR CLUSTER M13

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ABSTRACT

The luminosity function for M13 has been obtained down to the visual magnitude 19, by means of star counts on the plates taken with the 74-inch reflector of the Okayama Astrophysical Observatory. Particular attention has been paid to the counting method. A significant difference has been found between our luminosity function and the M3 luminosity function determined by Sandage. Our result for M13 gives a generally steeper gradient when compared with Sandage's result for M3.

By use of our luminosity function and the Population II stellar models given by Icko Iben, Jr., the lifetime and effective helium core mass have been obtained for each evolutionary stage up to the giant tip. Agreement between the present results and the theoretical ones is good, both for the low ($Y_e = 0.10$) and high ($Y_e = 0.35$) helium cases, contrary to the results obtained by Sandage for M3. Lifetimes for the horizontal-branch stars have been found to be 9×10^7 years for $X_e = 0.9$ and 5×10^7 years for $X_e = 0.65$. Some evidences which seem to favor the high helium content are discussed.

I. INTRODUCTION

The luminosity function of a globular cluster gives us important information concerning stellar evolution. It is particularly valuable because it is virtually determined by the behavior in the deep interiors of the stars. This is in contrast to the H-R diagram, which is strongly influenced by the convective envelope. However, extensive work on the luminosity function down to the main sequence, based on recent accurate photometry, is rather sparse, consisting only of the results by Sandage (1954*b*) for M3 and Tayler (1954) for M92.

Sandage used his luminosity function to obtain the time spent at each segment of the evolutionary track, and thus to determine the hydrogen-exhausted fraction of stellar mass by the so-called semi-empirical evolution method (Sandage 1954*a*, 1957*b*). However, these results showed a significant discrepancy with theoretical calculations in the sense that too much hydrogen consumption was found for $\log L/L_\odot \gtrsim 2.0$. One might suppose that this discrepancy could be attributed to the presence of stars in later evolutionary stages. If this were true, then another cluster that has different characteristics for later evolutionary stages might be expected to exhibit a luminosity function considerably different from that of M3. In M3 and M13, the horizontal branches, which presumably contain stars in later evolutionary stages, are considerably different; it therefore seems natural to suppose that their luminosity functions for $\log L/L_\odot \gtrsim 2.0$ ($M_v \lesssim 0.0$) are also different. Thus, we chose M13 as the comparison object, and took the plates through the observational seasons in 1962–1964.

II. OBSERVATIONAL MATERIAL AND STAR COUNTS

The series of plates was taken at the Newtonian focus of the 74-inch reflector of the Okayama Astrophysical Observatory, using a corrector. The plates used for the derivation of the luminosity function are listed in Table 1. All these are yellow exposures made

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on Eastman Kodak 103aD plates behind a Hoya Y50 filter. The scale of the plates is $22''.6/\text{mm}$.

Counts were made with reference to the series of standard stars. All the stars in a given field brighter than given standards were counted. We did not use the counts to the plate limit as the counting method, because trials showed a large scatter in the magnitude of the stars just visible. Sometimes this scatter amounted to 0.5 mag. On the other hand, if the stars were well over the plate limit, a difference of 0.1 mag could be seen by our method. A Nikon Shadowgraph Model 6C with a micrometric stand was used for counting. This equipment gave the precisely enlarged projection of the object on the stand to a viewing screen. The enlarging ratio was 10 for general cases, 20 for crowded fields. Special care was taken to keep the deviation from uniformity of surface brightness (on the viewing screen) to within 10 per cent. The coordinates of the plates were defined by the stars in Ludendorff's catalogue (Ludendorff 1905). The abscissa was given as the line through Nos. 120 and 1022, and the ordinate as the one through No. 526 and perpendicular to the abscissa.¹ The origin of this coordinate system was chosen to be the

TABLE 1
LIST OF M13 PLATES

Plate No. ^f	Date	Exp Time (min)	Approx Plate Limit
NS 207...	May 23, 1963	10	16 1
NS 208.	May 24, 1963	20	17 0
NS 209	May 25, 1963	20	17 5
NS 210	May 26, 1963	5	15 6
NS 294	May 6, 1964	90	20 3
NS 295	May 6, 1964	32	19 2
NS 306	May 7, 1964	20	18 0
NS 323.. . . .	May 11, 1964	3	16 7

same as the center of Arp's (1955) sectors which is about $20''$ southward from Ludendorff's origin. The directions were chosen to give Ludendorff's right ascension and declination axes. The zones to be counted were divided by concentric circles with radii $2'$, $3'$, $4'$, $6'$, and $8'$, and by 45° sectors which divided each quadrant in half. A half-transparent vinyl sheet attached to the screen was used for this purpose.

Counts in Arp's (1955) and Savedoff's (1956) catalogue, together with the eye estimate of magnitude for unmeasured stars, gave the data for $V \leq 16.0$ (and also ≤ 16.5 for the $4'$ – $8'$ zones). Some corrections were applied for Savedoff's magnitudes because of the existing errors in his results. These were made partly by measuring our plates, and partly by transferring his magnitudes, using our measurements. From $V \sim 16.5$ to $V \sim 18.0$, the standard stars for the counts were chosen from Savedoff's catalogue, at approximately 0.5-mag intervals. These standard stars were connected to the magnitude sequence by Baum, Hiltner, Johnson, and Sandage (1959) by measuring our plates. For the counts down to $V \sim 18.5$ and $V \sim 19.0$, it was necessary to seek the stars suitable for the counting standard. These stars were also calibrated by the stars of Baum *et al.*

Most of these magnitude calibrations were made by the Nikon Stellar Densitometer of the Okayama Astrophysical Observatory. Part of the work was also done by the iris photometer of the Astronomy Department, Columbia University. The mean deviations in the tabulated magnitudes with the same photometer reading are usually 0.04–0.05 mag. Thus, the magnitude errors for the counting standards probably do not exceed this

¹ Nos. 120, 1022, and 526 are III-11, I-78, and IV-5 in Arp's (1955) catalogue, respectively.

value. However, the magnitudes of the counting standards for $V \sim 16.0$, 16.5, 17.0, and 17.5 might be less certain due to the sparsity of standard stars given by Baum *et al.* and to the rather large field and plate errors existing in Savedoff's stars. These standards might be shifted to fainter magnitudes by amounts up to 0.1 mag. In summary, errors in the luminosity function due to the uncertainties in counting standards are not likely to exceed 10 per cent, except for the region between $V \sim 15.5$ and $V \sim 18$ where errors might exceed 20 per cent, since the magnitude intervals are taken to be about 0.5 mag. In addition, it is to be noted that the general trend in the luminosity function may be affected even less, as long as the magnitudes given by Arp (1955) and Baum *et al.* (1959) are correct, for the error at a particular magnitude standard affects only the luminosity

TABLE 2
STAR-COUNT DATA*

V	2'-3'	3'-4'	4'-6'	6'-8'	Plates Used
$\backslash 12\ 00..$	1	1	NS 208, 210, 306
$\backslash 12\ 50..$	5	4	1	1	
$\backslash 13\ 00..$	12	9	3	4	
$\backslash 13\ 50..$	25	12	6	5	
$\backslash 14\ 00..$	41	20	13	7	
$\backslash 14\ 50..$	62	32	24	14	
$\backslash 15\ 00..$	106	47	41	30	NS 295, 306
$\backslash 15\ 50..$	177	92	89	46	
$\backslash 16\ 00..$	227	127	134	70	
$\backslash 16\ 50..$	331 ± 15	178 ± 3	174	97	
$\backslash 16\ 52..$	456 ± 20	240 ± 10	249 ± 3	127 ± 4	
$\backslash 17\ 39..$		419 ± 18	355 ± 12	190 ± 7	NS 294, 295
$\backslash 17\ 58..$		870 ± 36	717 ± 38	340 ± 22	NS 294
$\backslash 18\ 38..$.	1159 ± 62	547 ± 36	NS 294
$\backslash 19\ 02..$.	2348 ± 104	1195 ± 52	NS 294

* The \pm numbers are the sum of the absolute values of the differences between the counts by the two authors in each divided zone.

function for two adjacent magnitude intervals. It is also found that our color system (Kodak 103aD + Hoya Y50) has a color equation such that

$$m_{pv,ours} \sim V - 0.1(B - V) + \text{const.}$$

However, this difference between m_{pv} and V hardly affects the derivation of the luminosity function.

The results of the counting are given in Table 2, together with the calibrated magnitudes. The plates used for counting always have a plate limit at least 1 mag fainter than that of the counting standards. A comparison of the counts in sectors shows the well-known ellipticity effect superposed by the effect of the shifted center. (After the counting, our center was found to be significantly different from the cluster center, but this has little effect on our purpose.) The counts down to $V = 16.5$ and fainter, except those for $V \leq 16.5$ and 4'-8', are averages of independent counts by the two authors. The agreement between our counts is generally good. This is evident from the \pm numbers given beside each count. These numbers are the sum of the absolute values of the differences between our counts in each divided zone. In view of these deviations given in Table 2, the errors in the luminosity function, which arise from the systematic difference in judging the brightness of the stars, do not seem to be much greater than 10 per cent.

III. LUMINOSITY FUNCTION

To obtain the luminosity function for the cluster from the star counts in Table 2, subtraction of the field stars is necessary. This is done by using tables by Seares, van Rhijn, Joyner, and Richmond (1925), with the correction for magnitude scales by Stebbins, Whitford, and Johnson (1950). Although the number of stars in each group within the outer zone of the cluster is appreciably reduced by this subtraction, the effect on the total numbers is quite small.

The other effect which must be taken into account is the difference in the central concentration for the stars with different magnitudes. This may be caused by the real mass segregation and/or the spurious background effect. The former presumably gives less concentration toward the center for stars with fainter magnitude, and the latter

TABLE 3
LUMINOSITY FUNCTION

V	M_v	Φ	Φ'^*	V	M_v	Φ	Φ'^*
11 9	-2 7	1		15 5	0 9	61	38
12 1	-2 5	2		15 7	1 1	54	37
12 3	-2 3	3		15 9	1 3	61	41
12 5	-2 1	4		16 1	1 5	77	53
12 7	-1 9	6		16 3	1 7	87	68
12 9	-1 7	6		16 5	1 9	102	88
13 1	-1 5	7		16 7	2 1	126	120
13 3	-1 3	8		16 9	2 3	167	151
13 5	-1 1	9		17 1	2 5	207	
13 7	-0 9	11		17 3	2 7	284	
13 9	-0 7	14		17 5	2 9	381	
14 1	-0 5	17		17 7	3 1	530	
14 3	-0 3	20		17 9	3 3	659	
14 5	-0 1	26		18 1	3 5	852	
14 7	+0 1	32	29	18 3	3 7	1050	
14 9	+0 3	43	35	18 5	3 9	1360	
15 1	+0 5	71	37	18 7	4 1	1720	
15 3	+0 7	71	39	18 9	4 3	2170	

* The luminosity function after subtraction of the horizontal-branch stars

affects the counts in the inner, crowded part of the cluster. However, the present data generally show no appreciable differences in central concentration. (This appears to be valid except for counts down to $V = 17.39$ and $V = 17.98$ for the 3'-4' zone, which seem to be affected by the background effect. Accordingly, these counts are not used for the derivation of the luminosity function.) Moreover, it will be shown in § IV that the mass difference throughout the whole giant branch is so small that it does not affect the results significantly. Thus, the luminosity function may be obtained directly from Table 2, although the counts for fainter magnitudes are rather limited in the outer portion of the cluster.

The luminosity function thus obtained is shown in Table 3, which gives the number of stars between $V + 0.1$ and $V - 0.1$ within the annular area which has radii of 2' and 8'. The numbers for $V \geq 17.1$ are obtained by applying a multiplication factor derived from the counts between $V = 15.00$ and 16.92. Also given in this table are the results after subtraction of the blue stars which form the continuous sequence from the horizontal branch (Arp and Johnson 1955; Savedoff 1956). According to King (1962), there is no difference in the degree of central condensation between the red and blue stars, so it is possible to obtain the true number ratio from catalogues of stars confined

to the outer portion of the cluster, e.g., from the catalogues of Arp and Savedoff. Arp's catalogue is used to separate the blue stars for $V \leq 15.5$, while Savedoff's is used for fainter magnitudes. The former gives a homogeneous star sample, but the homogeneity of the latter is somewhat questionable. Fortunately, however, the fraction itself of blue stars is rather small for $V > 15.5$, and thus the blue-star-subtracted luminosity function is not so much affected by this uncertainty. According to Savedoff's catalogue, there is a continuation of the blue sequence below the 17th magnitude, but the contribution to the total luminosity function is negligible because of the increasing number of stars on the main subgiant sequence. The total number of subtracted blue stars is 216. (As is well known, the horizontal branch of M13 consists almost entirely of the blue side of the RR Lyrae gap; this number may be taken as the total number of horizontal-branch stars.)

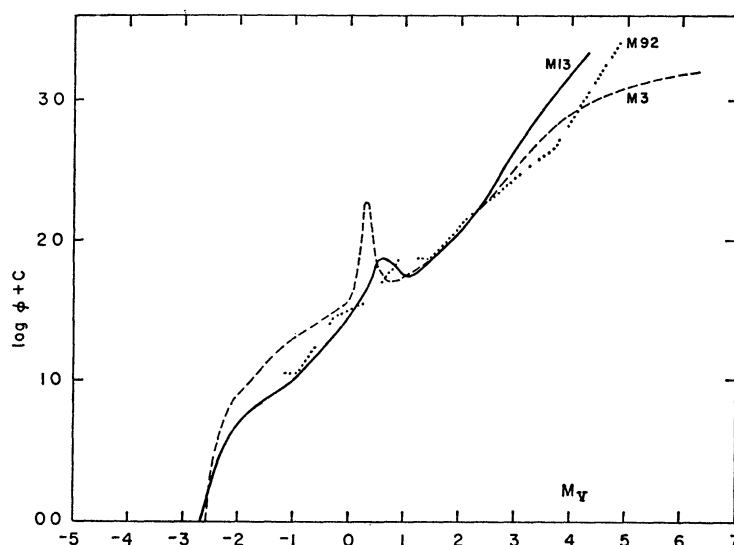


FIG. 1.—Luminosity functions for globular clusters. The ordinates for M3 and M92 are made to coincide at $M_v \sim 2$.

Although the number of blue stars in fainter magnitude segments may not be reliable, the total number of blue stars is probably fairly well determined since the contribution from the fainter magnitudes is rather small. The error may be less than 30 per cent.

Figure 1 compares our luminosity function for M13 with that for M3 by Sandage (1954*b*) and that for M92 by Tayer (1954). Arbitrary scale shifts in the ordinate are applied for M3 and M92 in order to see the differences among the clusters more clearly. The distance moduli for M3, M13, and M92 are taken to be 15.3, 14.6, and 14.6, respectively. These are almost equal to those given by Arp (1962, using reddening values by Kron and Mayall 1960) and Sandage (1964). Although considerable uncertainties exist in these values, they fortunately have little effect on the present comparison because the gradients of the luminosity functions are roughly the same for the brighter ($M_v \leq 0.0$) and fainter ($M_v \geq 1.5$) portions.

As shown by Figure 1 the difference between M3 and M13 is conspicuous, even apart from the region between $M_v = 0.0$ – 1.0 where the difference is due to the well-known variation in the horizontal branch. M13 has a generally steeper luminosity function than M3. In comparison with the subgiant stars ($M_v \sim 2$), which are adjusted to be the same by the shifts in the ordinate, M13 has fewer bright stars ($M_v \leq 0$) and more stars near the main sequence ($M_v \geq 3$) than M3. The difference in the brighter part cannot be explained by statistical fluctuation, because the χ^2 test, applied to stars down to $M_v = 2.32$ (by excluding those with $M_v = -0.1 \sim +1.4$), shows that this difference

occurs by chance, with a probability slightly less than 0.1 per cent. The occurrence by chance of the difference in fainter magnitude is more unlikely because of the larger star numbers.

The difference in the luminosity function in the brighter part may be mainly due to the difference in the number of stars which must be assigned to later evolutionary stages, such as the asymptotic branch stars. The present data (Arp 1955; Sandage and Walker 1966) seem to favor such interpretation, although the uncertainties are considerable.

The difference in the brighter part is just what we anticipated, at least qualitatively, but we did not expect the difference near the main sequence. However, it will be shown in § IV that our luminosity function for M13 is consistent with theoretical expectations. The difference between M3 and M13 is therefore surprising. New observations in M3 would be useful in confirming whether the deficit of faint stars does in fact exist.

As for M92, the luminosity function near the main sequence is inconsistent with theoretical expectations as is the case of M3. However, the data for the asymptotic branch of this cluster by Sandage and Walker (1966) again seems to favor our interpretation of the difference in the brighter part of the luminosity function.

IV. EVOLUTIONARY INTERPRETATIONS OF THE LUMINOSITY FUNCTION

The luminosity function obtained in § III is used to derive the lifetimes for each evolutionary stage by the same method that Sandage (1954*a*, 1957*b*) employed for M3. The necessary Population II stellar models near the main sequence were kindly given by Dr. Icko Iben, Jr. Model parameters are $M = 1.0 M_{\odot}$, $X_{\odot} = 0.90$ and $M = 0.7 M_{\odot}$, $X_{\odot} = 0.65$ (both for $Z_{\text{CNO}} = 10^{-4}$). Both models have turning points from the main sequence around $\log L/L_{\odot} = 0.33$. The homologous transformations are applied to these models to obtain luminosities and lifetimes for other masses. This may not cause significant error, because necessary changes in mass are found to be small. The exponent in the mass-luminosity relation is taken to be 4. The influence of the change of this exponent is small. The normal points for the H-R diagram and blanketing corrections are taken from the data by Sandage (1962). Bolometric corrections by Harris (1963) for $B - V \leq 1.5$ (luminosity class III) and by Allen (1963) for $B - V > 1.5$ (luminosity class I) are also used.

By fitting the turning point from the main sequence ($V = 18.5$) to Iben's models, the age of the cluster is found to be 1.69×10^{10} years for $X_{\odot} = 0.90$. By using this age, another point near $V = 18$ mag is brought back to the initial main sequence. The star number between these two points is compared with that on the corresponding initial main sequence in obtaining the scale factor by which the original luminosity function has to be multiplied. The original luminosity function used here is taken from Sandage (1957*a*, Table 2). One more point near $V = 19$ mag is also brought back to the initial main sequence and gives nearly the same multiplication factor. After that, the luminosity function in Table 3 can be used to bring each point back to the initial main sequence. The time spent in each magnitude segment is obtained as the difference between the age of a star now at the fainter edge of the segment and the age of a slightly more massive star (which is now at the brighter edge) evaluated for the epoch when it was at the same evolutionary stage as the former, using homology arguments. The same procedures are applied in the case of $X_{\odot} = 0.65$. The age is found to be 1.35×10^{10} years.

It is found from these procedures that the stars now on the evolutionary sequence from the turning point to the giant tip occupy a segment of width = 0.21 mag on the initial main sequence in the case of $X_{\odot} = 0.90$, and of width = 0.13 mag for $X_{\odot} = 0.65$. This smallness of magnitude ranges on the initial main sequence, that feeds the whole giant branch, is in conformity with the results by Sandage (1954*a*, 1957*b*), but is even more striking. The mass differences within the entire track are only 5 per cent for the low helium case and 3 per cent for the high helium case. The effects of these mass differences on the luminosity function of the whole cluster are calculated to be 17 and 5 per

cent, at the most. Therefore, the mass segregation may not significantly affect our results for the low helium case and has a negligible effect for the high helium case. In view of this smallness in the range of magnitude occupied by the stars on the initial main sequence, uncertainty in the original luminosity function has no appreciable effect on the results. In order to cause an appreciable effect, the original luminosity function must vary 20–30 per cent with $\Delta M_{\text{bol}} = 0.1\text{--}0.2$. This means that the gradient for the original luminosity function is similar to that for the present one, as can be seen easily from Figure 1; but this is improbable.

TABLE 4
LIFETIMES AND MASS FRACTIONS OF EFFECTIVE
HYDROGEN-EXHAUSTED CORES

V	$\log L/L_{\odot}$	$X_{\text{e}} = 0.90$		$X_{\text{e}} = 0.65$	
		$\Delta t(\text{years})$	M_{core}/M	$\Delta t(\text{years})$	M_{core}/M
<12.0	5×10^5	2×10^5
12.0	3.58	3×10^6	0.63	1.5×10^6	0.68
12.5	3.18	6×10^6	.56	3×10^6	.62
13.0	2.87	9×10^6	.495	4×10^6	.549
13.5	2.60	1.3×10^7	.447	7×10^6	.501
14.0	2.34	2.2×10^7	.409	1.1×10^7	.462
14.5	2.10	3.4×10^7	.373	1.7×10^7	.426
15.0	1.88	4.1×10^7	.339	2.1×10^7	.392
15.5	1.66	4.2×10^7	.314	2.1×10^7	.367
16.0	1.45	7.0×10^7	.299	3.5×10^7	.351
16.5	1.23	1.4×10^8	.283	6.9×10^7	.336
17.0	1.02	2.8×10^8	.265	1.4×10^8	.317
17.5	0.80	5.9×10^8	.242	3.4×10^8	.293
18.0	0.58		0.212		0.264

The uncertainty in the distance modulus hardly affects the relation between the luminosity and lifetime. Although the cluster age changes appreciably by the change in the distance modulus, the effect of this change on the luminosity-lifetime relation is roughly compensated by the change in the luminosity function for the same luminosity. The uncertainty in the turning point from the main sequence also has no appreciable effect on the luminosity-lifetime relation. In this case, the compensation for the change in the cluster age comes from the change in the luminosity function for the turning point. On the other hand, uncertainties existing in the blanketing and bolometric corrections will have some effect on the relation between the luminosity and the lifetime in the brighter stage, but probably not being much greater than 20 per cent, except at the very top of the giant sequence.

The resulting lifetimes are given in Table 4. The total lifetime for $V \leq 14.0$

($\log L/L_{\odot} \geq 2.34$) is 3.1×10^7 years for $X_e = 0.90$. This lifetime includes the one corresponding to the asymptotic branch stars, which has to be eliminated for the purpose of comparison with the theoretical lifetime for the pure giant stage before the helium flash. Based on an inspection of the H-R diagram by Arp (1955), the reduction factor required to obtain the lifetime for the pure giant stage is taken to be 0.8. Thus, the lifetime goes down to 2.5×10^7 years. Theoretical results for this helium content by Schwarzschild and others (Schwarzschild and Selberg 1962; Schwarzschild and Härm 1962; Härm and Schwarzschild 1964, 1966) and Hayashi, Hōshi, and Sugimoto (1962) are 2.5×10^7 and 2.2×10^7 years, respectively. Thus, the agreement is very good. For $X_e = 0.65$, a theoretical lifetime is not available, but the result by Hayashi *et al.* for $X_e = 0.61$ and $Z = 0.02$ may be used for comparison, since an inspection of the results by Härm and Schwarzschild (1964) shows that the heavy element content does not affect the lifetime in the giant stage. (This remark must be also applied in the case of $X_e = 0.90$, because our results are based on Iben's models with $Z_{\text{CNO}} = 10^{-4}$, while the results by Schwarzschild *et al.* and Hayashi *et al.* are for $Z = 10^{-3}$.) From Hayashi *et al.*, the lifetime for $\log L/L_{\odot} \geq 2.10$ is 1.9×10^7 years, and our result gives $2.6 \times 10^7 \times 0.8 = 2.1 \times 10^7$ years, again showing very good agreement.

Also given in Table 4 are the fractions of the star's mass, over which hydrogen has been exhausted at each magnitude. This is calculated by the formula

$$\frac{\Delta M_{\text{core}}}{M} = \frac{\langle L \rangle \Delta t}{E_{\text{H}} X_e M}$$

and summed up from the initial stage. Here E_{H} is the total amount of energy per gram liberated as radiation in hydrogen burning and assumed to be 6.15×10^{18} ergs. The total mass of the star is taken to be $1.06 M_{\odot}$ for $X_e = 0.90$ and $0.73 M_{\odot}$ for $X_e = 0.65$ as the mean mass between the star now at the giant tip and at the starting point of semi-empirical calculation ($V \sim 18$ mag). These mass fractions of the effective helium core are plotted by dashed curves in Figure 2 for $X_e = 0.90$ and in Figure 3 for $X_e = 0.65$. The dotted curves represent the reduced core mass fractions after a 20 per cent reduction of the star number for $V \leq 14.5$. This reduction eliminates the asymptotic branch stars, as mentioned before. Theoretical results by several authors are also represented as solid curves. The necessary transfers in mass are generally done by keeping the core mass constant, since it is well known that luminosity in the highly evolved stage is nearly determined by the helium core mass. Results from Iben's models for $\log L/L_{\odot} \lesssim 1.4$ are transferred by the same homology relation as before. Both for low and high helium content, the agreement between theory and our results is generally good. Discrepancies in the core mass fraction never exceed 0.10.

However, if the situation for the lower subgiant stage ($\log L/L_{\odot} \lesssim 1.0$) is more closely examined, the agreement is found to be much better for the case of $X_e = 0.65$. The gradient in Figure 2 for Iben's models (*solid curve*) changes abruptly around $\log L/L_{\odot} = 0.7$, going up by a factor of about 2 in comparison with the gradient for our semi-empirical result (*dashed curve*). There is no such abrupt change and subsequent discrepancy in Figure 3. (It is to be noted that this gradient is inversely proportional to the luminosity function.) However, this evidence for the high helium content of the stars in M13 seems to be far from established. Unfortunately, the luminosity function around this range ($V = 17$ – 18 mag) is rather ambiguous due to the possible errors in the magnitude of the counting standards, as mentioned in § II. It is also unknown whether Iben's models are correct in this respect. Nevertheless, it should be noted that, if Iben's models are correct, the helium content could be determined from a more accurately observed luminosity function around this magnitude range. More elaborate work both on the theoretical and observational side may be necessary. There are also some discrepancies between the theories and our observation in the giant stage ($\log L/L_{\odot} \gtrsim 2.0$), which are shown in Figures 2 and 3. These discrepancies occur in an opposite way

for the low helium case (Fig. 2) and high helium case (Fig. 3). Thus, this portion of the luminosity function might also be used for the determination of the helium content. But, at present, the situation seems to be not so hopeful because of many uncertainties inherent in the procedures, such as the blanketing and bolometric corrections or the separation of the stars in the later evolutionary stages.

The total number for the horizontal-branch stars given in § III can be used to obtain the lifetime for this branch. The results are 9.4×10^7 years for $X_e = 0.90$ and 4.7×10^7 years for $X_e = 0.65$. The theoretical results for $X_e = 0.90$, based on the double-energy-source stellar models, give the lifetime of $2-4 \times 10^7$ years (Nishida and Sugimoto 1962; Hayashi *et al.* 1962; Osaki 1963; Faulkner and Iben 1966). The uncertainty in the distance modulus affects our semi-empirical lifetimes; but theoretical lifetimes may also be affected in the same trend. The change in the turning point from the main sequence does not significantly change the lifetime, because the resulting change in the cluster age is roughly compensated by the change in the luminosity function for the turning point. Thus, in the case of $X_e = 0.90$, it is very difficult to explain the whole horizontal branch solely by models of this type. This difficulty, which was first pointed out by Nishida (see Osaki 1963), still remains valid. For $X_e = 0.65$, the situation seems to be less definitive. Although the models by Faulkner and Iben (1966) for this helium content give a longer lifetime than our result indicates, they do not seem to deny the possibility

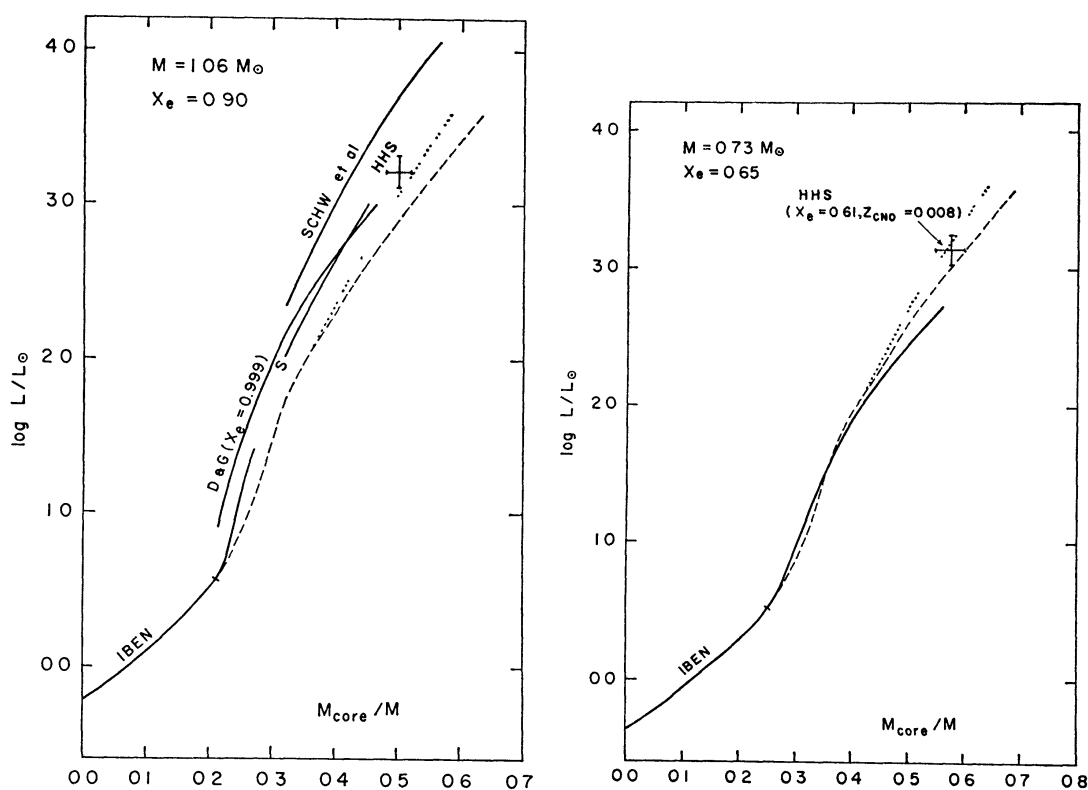


FIG. 2 (left).—Luminosity versus mass fraction of the effective helium core for the low helium case. Solid curves represent the theoretical results by the following authors: *D & G*: Demarque and Geisler (1963); *HHS*: Hayashi, Hōshi, and Sugimoto (1962), helium flash point with estimated errors; *IBEN*: Iben, private communication; *S*: Simoda (1961); *SCHW et al.*: A combination of the results by Schwarzschild and Selberg (1962), Schwarzschild and Härm (1962), and Härm and Schwarzschild (1964). Dashed curve gives our result by the semi-empirical method, starting from Iben's models at the point marked by a bar. Dotted curve expresses the result after subtracting the asymptotic branch stars.

FIG. 3 (right).—Luminosity versus mass fraction of the effective helium core for the high helium case. Curves and symbols have same meanings as in Fig. 2.

of the explanation of the total lifetime and the rough position on the H-R diagram for M13 horizontal branch by using the double-energy-source stellar models. More investigations from both the theoretical and observational side are necessary to clarify the point. It is particularly important to obtain a good luminosity function for the horizontal-branch stars down to the fainter magnitudes.

Finally, it may be interesting to discuss the situation for M3, in view of the good results for M13. The same calculations as applied for M13 are performed by using Sandage's luminosity function for M3 and Iben's models. The results show too much hydrogen consumption both for $X_{\odot} = 0.90$ and 0.65 . All the hydrogen fuel burns a little before the star attains the giant tip. This result may be about the same as that obtained by Woolf (1962*a, b*). To meet theoretical expectations up to $\log L/L_{\odot} \sim 2.0$, the M3 luminosity function near the main sequence should be increased by a factor of about 2. Therefore, if Sandage's luminosity function is correct, it is highly likely that some fundamental supposition in the semi-empirical evolution method does not hold, at least for M3.

For $\log L/L_{\odot} \gtrsim 2.0$, there still exist significant discrepancies between the semi-empirical and theoretical results even if one applies the reduction factor of 2 to the former, which comes from the supposed increase of the luminosity function near the main sequence as discussed above. These discrepancies may be attributed to the co-existence of stars in later evolutionary stages, such as the asymptotic branch stars, discussed in § III.

It is also possible to derive the total lifetime for the horizontal-branch stars of M3 from our semi-empirical calculations. The total number of horizontal-branch stars is estimated to be about 560. This number is obtained by extrapolating the detailed counts by Woolf (1964) to the cluster center using the integrated magnitude of Kron and Mayall (1960) and making allowance for the difference in central condensation for the RR Lyrae and yellow horizontal-branch stars. The error for this total number may be less than 10 per cent. The total lifetimes are 2.0×10^8 years for $X_{\odot} = 0.90$ and 1.2×10^8 years for $X_{\odot} = 0.65$. The effect of the uncertainties in the distance modulus and the turning point from the main sequence is not significant, as discussed in the case of M13.

In comparison, both Sandage (1957*b*) and Woolf (1964) have obtained the lifetime of 2.3×10^8 years. This is in good agreement with our result for $X_{\odot} = 0.90$, in spite of the difference in the cluster ages. As in the case of M13, the lifetime for $X_{\odot} = 0.90$ is too long when compared with the theoretical lifetimes based on the double-energy-source stellar models. For $X_{\odot} = 0.65$, the lifetime appears to be in good agreement with theoretical expectations by Faulkner and Iben (1966). However, one still has to deal with the serious difficulty near the main sequence, as mentioned above.

In conclusion, we would like to emphasize the important role of the luminosity function for the globular cluster as a powerful tool for the investigation of the evolution of the Population II stars, in view of the good results obtained for M13. There is also the possibility of determining the helium content by use of the subgiant portion of the luminosity function, as suggested before. Therefore, more extensive work for many clusters is highly desirable.

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Note added in proof on October 30, 1967.—After this paper went to press, a re-estimate of the lifetime for the bright stars in M13 was made, using models recently constructed by M. Simoda and Dr. Icko Iben, Jr. (to be published). The results for $V \leq 14.5$ ($\log L/L_{\odot} \geq 2.10$) are listed as follows.

LIFETIMES FOR $V \leq 14.5$ ($\log L/L_{\odot} \geq 2.10$)

Z	X	Lifetime (in units of 10^7 years)
10^{-3}	0 999	7 2
	9	5 9
	65	3 1
$5/3 \times 10^{-4}$	999833	7 0
	8	4 5
	0 65	2 7

As can be seen by comparing Table 4 and the above table, the lifetimes are essentially unaltered. Therefore, the good agreement between the theory and our luminosity function for M13 remains unchanged.

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